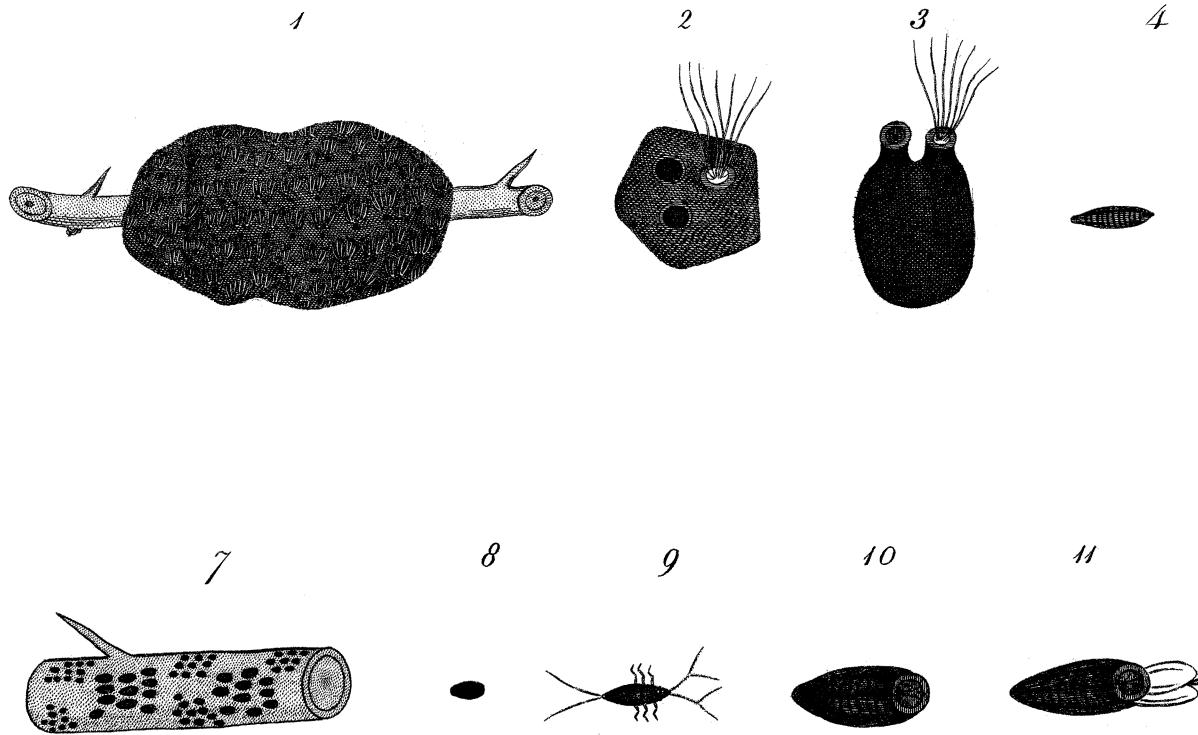


XVI. *The Longitudes of Dunkirk and Paris from Greenwich, deduced from the Triangular Measurement in 1787, 1788, supposing the Earth to be an Ellipsoid. By Mr. Isaac Dalby; communicated by Charles Blagden, M. D. Sec. R. S.*

Read May 19, 1791.

IN the account of the Trigonometrical Operation in 1787, 1788, which is given in the Philosophical Transactions, Vol. LXXX. after the distance of Dunkirk from the meridian of Greenwich has been determined on a parallel to the perpendicular at Greenwich, its longitude is found by spherical computation, on a supposition, that the surface of a sphere nearly coincides with that of the earth in an east and west direction, where the operation was performed ; and the magnitude of this sphere, or which amounts to the same thing, the value in parts of a degree, &c. of a measured arc on its surface (for as such the arc between the meridians of Botley Hill and Goudhurst may be considered) has been determined by actual observation at two stations nearly in the latitude of Dunkirk ; and this independent of any hypothesis which can sensibly affect the conclusion. The principles, though not strictly geometrical, admit of little objection ; and therefore, as much care was taken in observing the angles at these stations, upon which the directions of the meridians depend, the longitude of Dunkirk (and consequently that of Paris) as given in the Table, Vol. LXXX.





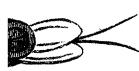
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p. 232. must be nearly true, whatever may be the real figure of the earth. But, it may be said, that the arc between the meridians of Botley Hill and Goudhurst ($17\frac{1}{2}$) is too short to infer from observation the value of the arc between the meridians of Greenwich and Dunkirk (amounting to near a degree and a half), sufficiently accurate for finding the longitude to great precision; because it has been remarked in the Appendix to the same Volume, that an error of $1''$, in either of the horizontal angles at the above stations, would cause a variation of near $6''$ of a degree in the longitude of Dunkirk or Paris.

M. BOUGUER's spheroid agreeing nearly with the meridional measurements, it was adopted for the purposes of latitude. But the degree perpendicular to the meridian in latitude $51^{\circ} 6' 53''$ is found to be 61248 fathoms (Vol. LXXX. p. 215.) which falls short of M. BOUGUER's degree about 22 fathoms; therefore, supposing the directions of the meridians to have been very accurately determined, the earth cannot be this spheroid, notwithstanding the ingenious hypothesis respecting the curve of the meridian. But it is also well known, that the measured degrees of latitude in different places are inconsistent with an elliptical meridian: for, suppose an ellipsoid to be determined with the degrees found at the equator and polar circle, the computed degrees in middle latitudes will be much longer than the measured ones, as it is well known; and the whole meridional arc between Greenwich and Paris will, on such an ellipsoid, exceed the measured arc by a quantity answering to about $21''$ of latitude. It is evident, however, that if we suppose small errors to have taken place in determining the celestial arcs, or differences of latitude in some of the operations (for there is little doubt but the terrestrial mensurations in general have been

been made exact enough), it will be easy to reconcile most of the results to an ellipsoid.

The following computations of the longitude are made on a supposition, that the earth is an ellipsoid, for the purpose of comparing the conclusions with what has been inferred from observation. It will be seen, that the ratio of the axes comes out very near the ratio assigned by Sir ISAAC NEWTON, or 229 to 230. It is determined of such a magnitude, by adhering nearly to the measured arc of the meridian between Greenwich and Paris, deduced from the late operation, that the computed meridional degrees differ but little from the measured ones in five different places in middle latitudes; but the defects at the equator and polar circle are supposed to be nearly equal to each other. This will be seen better by the following comparative view of the measured and computed degrees in the same latitudes.

<i>According to</i>	<i>Lat.</i>	<i>Measured.</i>	<i>Com-</i>	<i>Excess or de-</i>
		<i>Fath.</i>	<i>puted.</i> <i>Fath.</i>	<i>fect in mea-</i> <i>sured arc.</i>
M. CONDAMINE, &c.	0 0	60481	60344	+ 137
MASON and DIXON,	39 12	60628	60682	- 54
BOSCOVICH, &c.	43 0	60725	60738	- 13
CASSINI, &c.	45 0	60778	60768	+ 10
LIESGANIG*,	48 43	60839	60823	+ 16
French and English { Arc from latitude				
48° 50' 14" to } 160656 160662 - 6				
51° 28' 40" }				
MAUPERTUIS, &c.	66 20	61194	61057	+ 137

In the five comparisons, from latitude 39° 12' to Greenwich, the greatest error (54 fathoms) answers to about 3" of the celestial arc: neither of the other four differences amounts

* From the late Gen. ROY's Paper in the Phil. Trans. 1787.

to $1''$. The determination of M. BECCARIA is not brought into the comparison, because his measured degree in latitude $44^{\circ} 44'$ is longer than the measured one in latitude 45° .

The longitude of Dunkirk on this ellipsoid is found to be 9 m. 29.8 s. in time; and consequently that of Paris 9 m. $20\frac{1}{2}$ s., which is about $1\frac{1}{2}$ s. more than that inferred from the value of the measured arc between Goudhurst and the meridian of Botley Hill; and therefore the sum of the two horizontal angles at these stations would, on this ellipsoid, be only about $4''$ less than those found by actual observation.

Method of computation.

On an ellipsoid, where the degrees of the meridian at the equator and polar circle are 60481 and 61194 fathoms respectively, the degree in latitude $50^{\circ} 9'\frac{1}{2}$ (the middle latitude between Greenwich and Paris) will be 60981 fathoms, exceeding the measured degree by 140 fathoms (Vol. LXXX. p. 225.); therefore, if each of the former degrees was about 140 fathoms less, the computed and measured arcs in latitude $50^{\circ} 9'\frac{1}{2}$ would be nearly the same. But, that they also may nearly agree in latitude 45° , let the degrees at the equator, and in latitude $50^{\circ} 9'\frac{1}{2}$, be taken 60344 and 60844; then, from these two degrees, the ratio of the axes will be found as the tangents of the arcs $50^{\circ} 9'\frac{1}{2}$ and $50^{\circ} 1' 35'\frac{1}{2}$; and the semi-axes 3489932 and 3473656 fathoms *.

The

* Determined thus: If right lines are drawn perpendicular to the curve of a conic section to meet the axis, it is known, that the radii of curvature at the points in the curve from whence these lines are drawn, will be as the cubes of these lines. Hence, if PC, GB, EC (Tab. VII. fig. 1.), are perpendicular to the curve, the radii of curvature

The length of the whole meridional arc between Greenwich and Paris on this ellipsoid is six fathoms greater than the measured arc; the degree in latitude $48^{\circ} 43'$, 16 fathoms less; in latitude 45° , 10 fathoms less; in 43° , 13 fathoms greater; and that in latitude $39^{\circ} 12'$, 54 fathoms greater. The degrees at the equator and polar circle are considerably less than the measured ones, conformable to the hypothesis.

Suppose CE, CP (fig. 1.), are the greater and less semi-axes of the ellipsoid; G Greenwich; PGE its meridian; PD the meridian of Dunkirk; and let GBA be perpendicular to the curve of the meridian at G; then GA will be the shorter axis of the elliptical section which is the perpendicular to the

at P, G, E, will be as PC^3 , GB^3 , and $\left(\frac{PC}{CE}\right)^3$, because at the point E (or equator) the line so drawn will become the radius of curvature itself, or $\frac{PC^2}{CE}$. There-

fore $GB^3 : \left(\frac{PC^2}{CE}\right)^3 :: \text{rad. curv. at } G : \text{rad. curv. at } E :: \text{length of a deg. in the lat. of } G : \text{length of a deg. at } E$, the equator. Let the arc ERL be described with the radius CE; draw CR parallel to GB, RS parallel to PC, and join CK; then, by the nature of the ellipse, $CR(CE) : CK :: GB : \text{half the parameter, or } \frac{CP^2}{CE}$; therefore $CE^3 : CK^3 :: GB^3 : \left(\frac{CP^2}{CE}\right)^3 :: 60844 : 60344$ (supposing the lat.

of the point G to be $50^{\circ} 9\frac{1}{2}'$), or $CE(CR) : CK :: \sqrt[3]{60844} : \sqrt[3]{60344}$; but $CR : CK :: \text{fine SKC} : \text{fine KRC (co-lat.)}$; therefore, $\frac{\sqrt[3]{60844}}{\sqrt[3]{60344}} \times \text{cofine lat.} =$

fine SKC; hence the angle SCK is given ($50^{\circ} 1' 35\frac{1}{2}''$); therefore, as $\text{tang. SCK} : \text{tang. lat. (SCR)} :: \text{SK} : \text{SR} :: \text{lesser semi-axis CP} : \text{greater CE}$. And putting $d = 57.295779$, &c. the degrees in the circular arc which is equal to the radius we have

$\frac{\text{tang. lat.}}{\text{tang. SCK}}^2 \times 60344 d = 3489932$ fathoms the longer semi-axis; and

$\frac{\text{tang. lat.}}{\text{tang. SCK}} \times 60344 d = 3473656$, the shorter.

meridian

meridian at Greenwich, and the angle EBG will be the latitude of Greenwich, or $51^{\circ} 28' 40''$. Let HO (parallel to GA) be the section of the parallel to that perpendicular, passing through Dunkirk. Then by the Table, p. 232. (Vol. LXXX.) the arc GH is 152549 feet; but this arc exceeds the real distance of the parallels GA, HO, not more than a fathom; therefore this distance may be taken = 25424 fathoms. Now the sections GA, HO, of the ellipsoid being similar, from the known properties of the figure, we shall get HO the shorter axis of the section of the parallel = 6959396, its longer axis = 6979374, and HW = 3531757 fathoms, W being the point where HO cuts the axis PI of the ellipsoid. Hence, if D be Dunkirk, and the arc HD the measured arc of the parallel, we have given the length of this arc, or 547058 feet (Table, p. 232.) = 91176 fathoms, and also the point W in the lesser axis of the section HO, to determine the angle HWD in the plane of this section. But reverting the series which exhibits the length of an elliptical arc in terms of the *absciss* and *ordinate*, will be of little use in the present case, where the arc and its chord are very near of the same length: For, let HKOL (fig. 2.) be the section of the parallel, where HO = 6959396, and KL = 6979374, are the axes; and HW = 3531757, as in fig. 1.; also, suppose HS is the radius of curvature at H, or at the middle of HD; then, if we conceive the arc HD to be a right line, or described with the radius HW, or with HS (3499700) and thence determine the angle SWD from the two sides SD, SW, and the included angle (the supplement of HSD); in either case we get the angle HWD the same, or $1^{\circ} 28' 44'',8$ to within $1''$. This angle being obtained, the inclination of the planes PHW, PDW (the planes of the meridians of

Greenwich and Dunkirk, fig. 1.), or the longitude of the point D, will be found by the common proportion which in a right-angled spherical triangle determines an angle when the legs are given: this will be obvious by conceiving a sphere (of any magnitude) to be described about W as a center.

Hence, as $\text{rad.} : \text{cotang. angle HWD } (1^\circ 28' 44'' .8) :: \text{fine}$
 $\text{angle HWP } (38^\circ 31' 20'') : \text{cotang. } 2^\circ 22' 26'' \frac{1}{2}$, the inclination of the planes of the meridians PH, PD, or longitude of Dunkirk on this ellipsoid. And as the difference of meridians of Paris and Dunkirk is $2' 21'' .9$ (for this will not be materially affected by different hypotheses) the longitude of Paris will be $9 \text{ m. } 20 \frac{1}{2} \text{ s.}$ in time. The longitude of Dunkirk from Paris ($2' 21'' .9$) is the mean longitude deduced at p. 223. (Vol. LXXX.), which is only $1''.1$ less than that given in the *Connoissance des Temps*, 1788.

The method of computing the latitude of the point D (was it necessary) is thus: as $\text{rad.} : \text{cosine DWH} :: \text{cosine HWP} : \text{cosine DWP}$; and since the point W in the axis FW is given, and also the angle DWP in the plane of the meridian PD (by the foregoing proportion), the point D will be determined by the properties of the ellipse; which in fact is nothing more than finding the inclination of the vertical at the point D with the given line DW, which inclination added to the angle DWP, gives the co-latitude of the point D. And hence may be evinced the truth of what is advanced at p. 199. (Vol. LXXX.), that if the value of an arc on a spheroid, considered as an arc of a great circle perpendicular to the meridian, be given, the longitude may be found by spherical computation, but not the latitude. For conceive the arc HD to be perpendicular to the meridian at H, then the angle HWP would be the co-latitude

of the point H; and the former proportion would give the longitude of D, whether the figure was a sphere or spheroid; and the angle DWP (found by the latter proportion) would be the co-latitude of D supposing it a sphere, in which case the point W becomes the center; but this will not hold in a spheroid, because DW would not be perpendicular to the meridian at D.

The forgoing method of computing the longitude from the measured arc of a parallel on a given ellipsoid (though evidently the direct one), will be tedious, especially when the lengths of the measured arcs (GH, HD) are very considerable. But when the latitude of the point H is determined from the measured arc GH (on the known meridian), and the extent of the other arc (HD), or rather the angle HWD, is not more than two or three degrees, the same conclusions, extremely near, may be obtained in the following manner, which is nearly the same as the method used in computing the longitudes in the Table of General results, p. 232. (Vol. LXXX.).

Suppose G and D (fig. 1.) to be Greenwich and Dunkirk; PH, PD, their meridians, as before; and let HD (instead of its being a parallel to the perpendicular at Greenwich) be an arc of an ellipse cutting the meridian of Greenwich at right angles, suppose in the point H. Then the arc GH being = 152549 + 50 feet nearly (because the ellipse which passes through D, and is at right angles to the meridian PG, will fall about 50 feet to the south of the point cut by the parallel), therefore the value of the arc GH, or 25433 fathoms, will, on this ellipsoid, be $25' 4''.4$, and consequently the angle PWH, or the co-latitude of H, is $38^\circ 56' 24''.4$. Now, the radius of curvature of this perpendicular ellipse at H, the extremity of its lesser

axis, will be 3499798 fathoms*, which, divided by 57.295779, &c. (the degrees in the circular arc which is equal to the radius), gives 61083 fathoms for a degree on this ellipse, considered as a great circle perpendicular to the meridian at the point H on the ellipsoid; and since the length of this arc (HD) will be nearly the same as that of the parallel, or 91176 fathoms, its value will be $1^{\circ} 29' 33''.6$ (the arc DH, or rather the angle DWH). Hence, as $\text{rad. : cotang. } 1^{\circ} 29' 33''.6 \text{ (HWD)} :: \text{fine}$ $38^{\circ} 56' 24''.4$ (HWP) : cotang. $2^{\circ} 22' 26''.8$, the longitude of D, or Dunkirk, the same as before, very near; hence the longitude of Paris will be $2^{\circ} 20' 4''.9$. But the same may be obtained from the mean distance of the meridians of Greenwich and Paris, or 537950 feet. See p. 599. in the Appendix to Vol. LXXX.

It appears from the foregoing hypothesis, that the measured degrees of the meridian in middle latitudes will answer nearly on an ellipsoid whose axes are in the ratio assigned by Sir Isaac NEWTON. But this will receive further confirmation from the fifth ellipsoid in the second Table, p. 232. (Vol. LXXX.), where the near agreement between the computed and measured arc of the meridian between Greenwich and Perpignan (differing but about 52 fathoms in the extent of $8^{\circ} 46' 44''$) would be somewhat extraordinary, were we certain that the latitude of Perpignan ($42^{\circ} 41' 56''$) is correct; but this is suspected by M. DE LA

* It is not necessary to determine the axes of this ellipse, because when HW is perpendicular to the curve of the meridian, it will (by the nature of the figure) be the radius of curvature of the arc HD at the point H. Hence, if we put v for the cotang. and c for the cosine of the latitude of the point H, and let a denote the fine of an arc whose tang. is $\frac{CE}{CP} \times v$; then $\frac{a}{c} \times CE = HW$, by the properties of the ellipse.

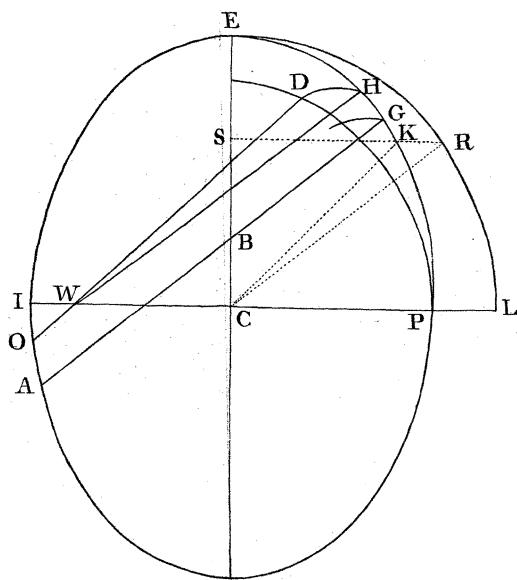


Fig. 1.

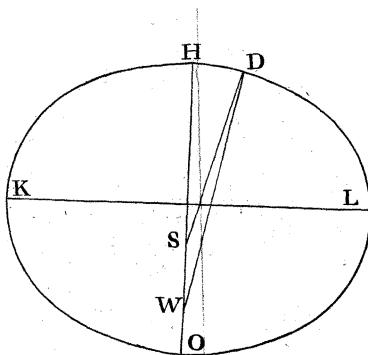


Fig. 2.

CAILLE. See *Mem. de l'Acad.* 1758. The computed arc, however, between Greenwich and Paris is 19 fathoms longer than the measured arc, which answers to a little more than 1" of latitude.

The longitude of Paris on this ellipsoid is 9 m. $20\frac{4}{10}$ s.

If it be contended, that the operations at the equator and polar circle were as correct as those executed for the like purpose in middle latitudes; and that a kind of mean between the extreme results ought to be preferred; we shall still get an ellipsoid, whose axes are nearly as 229 to 230, by taking the degrees at the equator and polar circle each 70 fathoms less, and that in latitude $50^{\circ} 9' \frac{1}{2}$ as much greater than the measured ones; and the longitude of Paris will be found 9 m. $19\frac{7}{10}$ s. But the computed meridional arc between Greenwich and Paris will exceed the measured one by a quantity answering to about 11" of latitude.

It is almost needless to observe, that the longitude of Paris (9 m. 20 s.) deduced by Dr. MASKELYNE from the different results found by astronomical observations (*Phil. Trans.* 1787) agrees to less than half a second with either of the above determinations.

N. B. In the Table of General Results, p. 232. Vol. LXXX. read $18' 46''$ for the longitude of Wrotham Hill; this should have been corrected in the Appendix.



